Mašinski fakultet

(kolokvijum)

(priprema za ispit) ①

1) Izračunati neodređene integrale:

a) $\int \frac{4x-3}{x^2-2x+5}\,dx$ b) $\int \frac{dx}{x^2-6x+18}$ c) $\int \frac{dx}{\sqrt{6x-x^2}}$

e) $\int x\cos(2x)\,dx$ f) $\int (3x^2+6x+5)\,\mathrm{arctg}\,x\,dx$

g) $\int \frac{dx}{\sqrt{8x-x^2}}$ h) $\int \frac{4x-1}{\sqrt{x^2+2x-2}}\,dx$

Rj.

a) $x^2-2x+5=(x-1)^2+4$ | $4x-3=4(x-\frac{3}{4})=$

$x^2-2x+5=t \;/'$ | $=4\cdot(x-1+1-\frac{3}{4})=$

$(2x-2)\,dx=dt$ | $=4(x-1)+1$

$2(x-1)\,dx=dt$

$(x-1)\,dx=\frac{dt}{2}$ (*)

$$I=\int \frac{4(x-1)+1}{x^2-2x+5}\,dx=4\int\frac{x-1}{x^2-2x+5}+\int\frac{dx}{x^2-2x+5}=$$

$$=4I_1+I_2$$

I_1: smjena $x^2-2x+5=t \stackrel{(*)}{\Rightarrow} I_1=\frac{1}{2}\int\frac{dt}{t}=\frac{1}{2}\ln|x^2-2x+5|+c$.

$$I_2 = \int \frac{dx}{(x-1)^2+4} = \int \frac{dx}{4\left(\left(\frac{x-1}{2}\right)^2+1\right)} = \frac{1}{4}\int \frac{dx}{\left(\frac{x-1}{2}\right)^2+1} =$$

$$= \ulcorner \frac{x-1}{2} = t \,/\, ' \qquad \frac{dx}{2} = dt \Rightarrow dx = 2dt \lrcorner = \frac{1}{4}\cdot 2 \cdot \int \frac{dt}{t^2+1} =$$

$$= \frac{1}{2}\operatorname{arctg}\left(\frac{x-1}{2}\right) + c.$$

$$I = 4I_1 + I_2 \ldots$$

b) $$\int \frac{dx}{x^2-6x+18} = \ulcorner x^2-6x+18 = (x-3)^2 - 9 + 18 = (x-3)^2+9 \lrcorner =$$

$$= \int \frac{dx}{(x-3)^2+9} = \int \frac{dx}{9\left(\left(\frac{x-3}{3}\right)^2+1\right)} = \frac{1}{9}\int \frac{dx}{\left(\frac{x-3}{3}\right)^2+1} =$$

$$= \ulcorner \frac{x-3}{3} = t \qquad \frac{dx}{3} = dt \lrcorner = \frac{1}{3}\int \frac{dt}{t^2+1} = \frac{1}{3}\operatorname{arctg}\left(\frac{x-3}{3}\right) + c.$$

c) $$\int \frac{dx}{\sqrt{6x-x^2}} = \ulcorner 6x - x^2 = -(x^2-6x) = -\left((x-3)^2-9\right) = 9-(x-3)^2 \lrcorner =$$

$$= \int \frac{dx}{\sqrt{9\left(1-\left(\frac{x-3}{3}\right)^2\right)}} = \frac{1}{3}\int \frac{dx}{\sqrt{1-\left(\frac{x-3}{3}\right)^2}} = \ulcorner \frac{x-3}{3} = t \qquad \frac{dx}{3} = dt \Rightarrow dx = 3dt \lrcorner$$

$$= \frac{1}{3}\cdot 3 \int \frac{dt}{\sqrt{1-t^2}} = \arcsin\left(\frac{x-3}{3}\right) + c.$$

e) $\int x\cos(2x)\,dx = \begin{bmatrix} x=u & \cos(2x)\,dx=dv \\ dx=du & v=\frac{1}{2}\sin 2x \end{bmatrix} =$

$= x\frac{1}{2}\sin 2x - \frac{1}{2}\int \sin(2x)\,dx = \frac{x}{2}\sin(2x) + \frac{1}{2}\frac{1}{2}\cos(2x) + C.$

f) $\int (3x^2+6x+5)\,\mathrm{arctg}\,x\,dx = \begin{bmatrix} \mathrm{arctg}\,x = u & (3x^2+6x+5)dx=dv \\ \frac{dx}{1+x^2}=du & v=x^3+3x^2+5x \end{bmatrix}$

$= (x^3+3x^2+5x)\,\mathrm{arctg}\,x - \underbrace{\int \frac{x^3+3x^2+5x}{x^2+1}\,dx}_{I_1}$

I_1:
$(x^3+3x^2+5x):(x^2+1) = x+3 \Rightarrow$
$\underline{-x^3 \mp x}$
$3x^2+4x$
$\underline{-3x^2 \mp 3}$
$4x-3$

$I_1 = \int (x+3)\,dx + \int \frac{4x-3}{x^2+1}\,dx =$

$= \frac{x^2}{2} + 3x + I_2$

I_2 (sami, slično kao a))

g) i h) sami.

2) Izračunati P figure ograničene grafikom fje $y=-x^2+7x-10$ i njenom tangentom u t. $A(3,y_0)$ i Ox-osom.

Rj

$y=-x^2+7x-10 \to$ parabola

$-x^2+7x-10=0$

$$x_{1,2}=\frac{-7\pm\sqrt{49-40}}{-2}=\frac{-7\pm3}{-2}$$

$x_1=2$ $\quad$ $x_2=5$

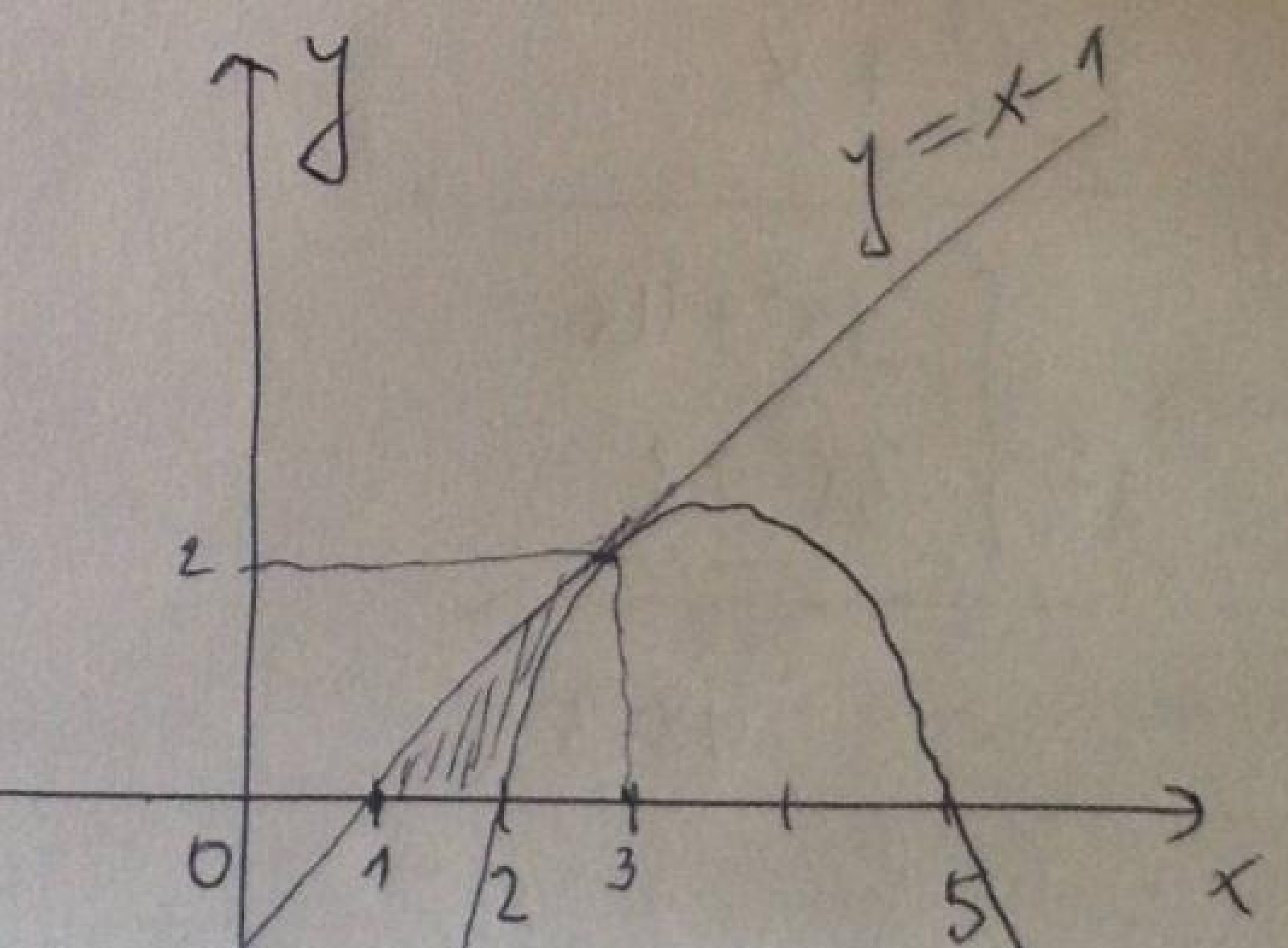
t: $y-y_0=k_t(x-x_0)$ $\qquad$ $k_t=y'(A)$

y_0: $y_0=-9+7\cdot3-10$ $\qquad$ $y'=-2x+7\Big|_{A(3,2)}=1$

$y_0=2 \Rightarrow A(3,2)$

t: $y-2=x-3$

$\boxed{y=x-1}$



x	0	1
y	-1	0

$$P=\int_1^3 (x-1)\,dx-\int_2^3(-x^2+7x+10)\,dx=$$

$$=\left(\frac{x^2}{2}-x\right)\Big|_1^3-\left(-\frac{x^3}{3}+\frac{7x^2}{2}+10x\right)\Big|_2^3=\left(\frac{9}{2}-3\right)-\left(\frac{1}{2}-1\right)-\left[\left(-9+\frac{7\cdot9}{2}+30\right)-\left(-\frac{8}{3}+14+20\right)\right]$$

3) Naći P figure ograničene grafom fje $y = 3\ln(2-x)$ (3)
i njenom normalom u tački $A(2-e^2, y_0)$ i Ox-osom

Rj:

$y = 3\ln(2-x)$; $2-x>0$

$-x>-2$

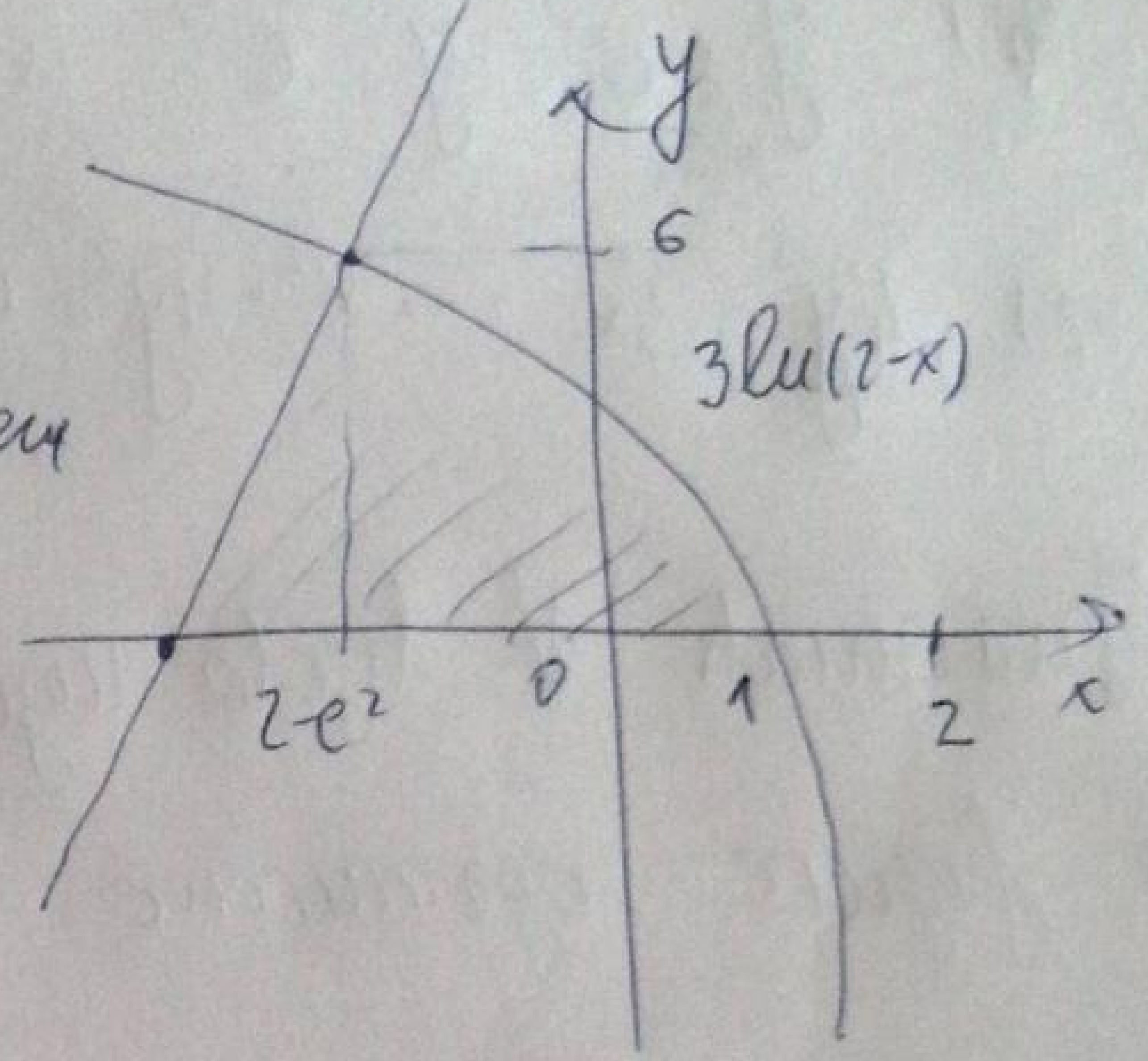
$\boxed{x<2}$ domen

$0 = 3\ln(2-x)$

$\ln(2-x)=0$

$2-x=1$

$\boxed{x=1}$



$n: \; y-y_0 = k_n(x-x_0)$

$n: \; y-6 = k_n(x-2+e^2)$

$y_0 = 3\ln(\cancel{2}-\cancel{2}+e^2)$

$y' = \frac{-3}{2-x}\Big|_{A(2-e^2,6)} = \frac{-3}{\cancel{2}-\cancel{2}+e^2} = \frac{-3}{e^2}$

$y_0 = 3\ln e^2$

$\boxed{y_0=6} \Rightarrow \underbrace{A(2-e^2,6)}$

$\underbrace{k_n = \frac{e^2}{3}}$

$n: \; y-6 = \frac{e^2}{3}(x-2+e^2)$

$\underbrace{y = \frac{e^2}{3}x - \frac{2e^2}{3} + \frac{e^4}{3} + 6}$

$y=0 \Rightarrow \frac{e^2}{3}x = \frac{2e^2}{3} + \frac{e^4}{3} + 6$

$e^2 x = 2e^2 + e^4 + 18$

$\underbrace{x = 2+e^2+\frac{18}{e^2}}$

$P = \int_{2+e^2+\frac{18}{e^2}}^{2-e^2}\left(\frac{e^2}{3}x - \frac{2e^2}{3} + \frac{e^4}{3} + 6\right)dx + \int_{2-e^2}^{1} 3\ln(2-x)dx$

4) Izračunati P figura ograničenih sa:

a) $x^2+y^2=16$; $y^2=4(x+1)$, $x\in[-1,4]$

b) $y=e^x$, $y=e^{-x}$, $x=2$ i Oy-osom

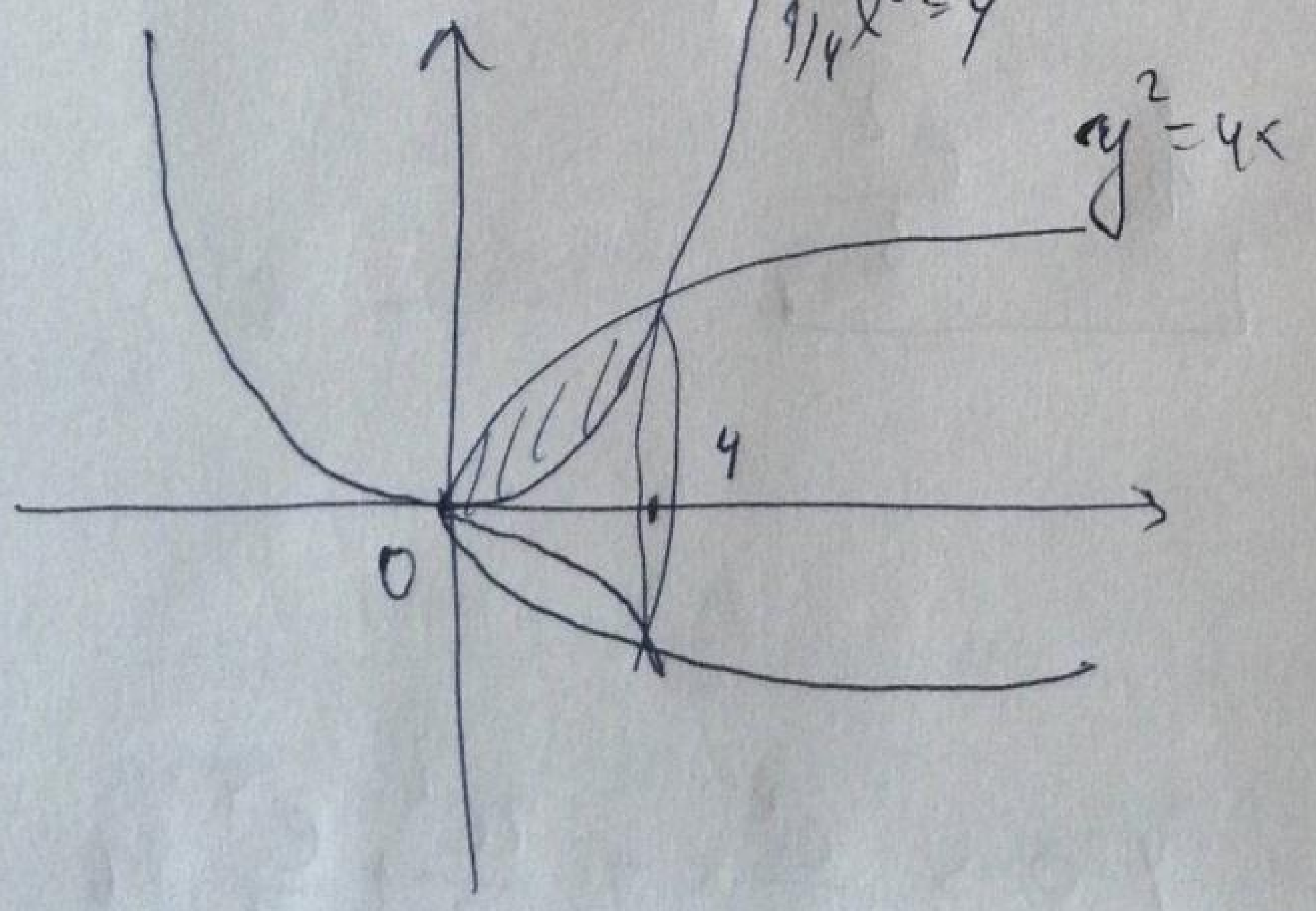
c) $4y^2=3x$; $x^2+4y^2=4$ $x\in[0,2]$

5) Izračunati V tijela koje nastaje rotacijom oko ose Ox figure ograničene krivim $4y^2=x^2$ i $y^2=4x$

Rj

$4y=x^2$ $\qquad$ $y^2=4x$

$y=\frac{1}{4}x^2$ (parabola) $\qquad$ parabola



Presjek: $\begin{cases} y=\frac{x^2}{4} \\ y^2=4x \end{cases}$

$\frac{x^4}{16}=4x \;/\cdot 16$

$x^4=16\cdot 4x$

$x(x^3-64)=0$

$x=0 \lor x^3=64$

$\boxed{x=4}$

$$V=\pi\int_0^4 4x\,dx-\pi\int_0^4\left(\frac{1}{4}x^2\right)^2dx =$$

$$=\overset{2}{\cancel{4}}\pi\frac{x^2}{\cancel{2}}\Big|_0^4-\pi\frac{1}{16}\frac{x^5}{5}\Big|_0^4=2\pi\cdot16-\frac{\pi}{80}\cdot4^5=32\pi-\frac{16}{5}\pi$$

6) Ispitati konvergenciju br. redova:

4

a) $\sum_{n=1}^{\infty} \frac{(n+1)^2}{9^{n+1}}$ b) $\sum_{n=1}^{\infty} \frac{1\cdot 5\cdot 9\cdots(4n-3)}{2\cdot 5\cdot 8\cdots(3n-1)}$

c) $\sum_{n=1}^{\infty} \frac{2\cdot 5\cdot 8\cdots(3n-1)}{1\cdot 5\cdot 9\cdots(4n-3)}$ d) $\sum_{n=1}^{\infty} \frac{1}{4^n}\left(\frac{2n}{2n+1}\right)^{2n^2}$

e) $\sum_{n=1}^{\infty} \left(\frac{4n}{9n+5}\right)^{7n}$ f) $\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdots(2n-1)}{4^{2n+1}\cdot n!}$

g) $\sum_{n=1}^{\infty} \left(\frac{3n^2+4n}{3n^2+4n-1}\right)^{4n^3}$ h) $\sum_{n=1}^{\infty} \frac{1}{(5n^2+4)^2}$ i) $\sum_{n=1}^{\infty} \frac{\sqrt{16n^2}-\sqrt{16n^2-1}}{n^4}$

Rj

a) $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+2)^2}{\cancel{9^{n+2}}}}{\frac{(n+1)^2}{\cancel{9^{n+1}}}} = \lim_{n\to\infty} \frac{(n+2)^2}{9(n+1)^2} = \frac{1}{9} < 1 \rightarrow$

Po Dalamb. krit. polazni red konv.

b) $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{\cancel{1\cdot 5\cdot 9\cdots(4n-3)}(4n+1)}{\cancel{2\cdot 5\cdot 8\cdots(3n-1)}(3n+2)}}{\frac{\cancel{1\cdot 5\cdots(4n-3)}}{\cancel{2\cdot 5\cdots(3n-1)}}} = \frac{4}{3} > 1 \rightarrow$

Po Dalamb. krit polazni red konv

d) $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{1}{4}\left(\frac{2n}{2n+1}\right)^{2n} = \frac{1}{4}\lim_{n\to\infty}\left(1+\frac{\cancel{2n}-\cancel{2n}-1}{2n+1}\right)^{2n} =$

$$= \frac{1}{4}\lim_{n\to\infty}\underbrace{\left[\left(1+\frac{-1}{2n+1}\right)^{\frac{2n+1}{-1}}\right]}_{e}^{\frac{-1}{2n+1}\cdot 2n} = \frac{1}{4}e^{-2} = \frac{1}{4e^2} < 1$$

$\Rightarrow$ Po Košijevom krit konv

e) $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty}\left(\frac{4n}{9n+5}\right)^7 = \left(\frac{4}{9}\right)^7 < 1 \Rightarrow$ Po koš. krit konv

h) $\sum_{n=1}^{\infty} \frac{1}{(5n^2+4)^2}$ $\quad a_n = \frac{1}{25n^4+40n^2+16}$

Posmatramo red $b_n = \sum \frac{1}{n^4}$

$\sum_{n=1}^{\infty} \frac{1}{n^4}$ je konv jer $4>1$

$$\Rightarrow \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{25n^4+40n^2+16}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{n^4}{25n^4+40n^2+16} = \frac{1}{25} \neq 0$$

$\Rightarrow$ Po poredb. krit. i $\sum_{n=1}^{\infty} \frac{1}{(5n^2+4)^2}$ je konv

i) $a_n = \frac{\sqrt{16n^2}-\sqrt{16n^2-1}}{n^4}\cdot\frac{\sqrt{16n^2}+\sqrt{16n^2-1}}{\sqrt{16n^2}+\sqrt{16n^2-1}} =$ (5)

$$= \frac{16n^2-16n+1}{n^4\left(\sqrt{16n^2}+\sqrt{16n^2-1}\right)} = \frac{1}{n^4\left(\sqrt{16n^2}+\sqrt{16n^2-1}\right)}$$

Posmatramo red sa opštim članom $b_n = \frac{1}{n^4\cdot n}$

$\sum_{n=1}^{\infty}\frac{1}{n^5}$ je konv. jer $5>1$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\cancel{n^4}\cdot n}{\cancel{n^4}\left(\underbrace{\sqrt{16n^2}}_{4n}+\sqrt{16n^2-1}\right)} = \frac{1}{8}\neq 0$$

$\Rightarrow$ Polazni red je konv. po poredb. krit.

7) Ispitati konv. funkcionalnih redova:

a) $\sum_{n=1}^{\infty}\frac{2\sin(nx)}{3n^5+1},\ x\in\mathbb{R}$

b) $\sum_{n=1}^{\infty}\frac{4\cos\cancel{(nx)}\sin(nx)}{2n^3+5},\ x\in\mathbb{R}.$

Rj:

a) $\frac{2\sin(nx)}{3n^5+1} \underset{\uparrow}{\leq} \frac{2\cdot 1}{3n^5+1} = \frac{2}{3n^5+1}$

$\sin(nx) \leq 1$

Brojni red $\sum \frac{1}{n^5}$ je konv. $\Rightarrow$

polazni fjaralni red je konv po Vej. t.

b) slicno : $\sin(nx) + 4\cos(nx) \leq 5$

$\circledast$ napomena: $\operatorname{arctg} x \in (-\pi/2, \pi/2)$

$\operatorname{arcctg} x \in (0, \pi)$

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1. Funkciju $f(x) = e^{ax}$ razviti u Furijeov red na intervalu $(-\pi, \pi)$.

Rešenje:

$$l = \frac{\pi - (-\pi)}{2} = \pi \qquad a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{ax}dx = \frac{1}{\pi a}e^{ax}\Big|_{-\pi}^{\pi} = \frac{1}{\pi a}\left(e^{ax} - e^{-ax}\right)$$

$$a_n = \frac{1}{\pi}\underbrace{\int_{-\pi}^{\pi} e^{ax}\cos nx\, dx}_{I} = \frac{a(-1)^n}{\pi(n^2+a^2)}\left(e^{a\pi} - e^{-a\pi}\right)$$

$$I = e^{ax}\ \frac{\sin nx}{n}\Big|_{-\pi}^{\pi} - \frac{a}{n}\int_{-\pi}^{\pi} e^{ax}\sin nx\, dx = -\frac{a}{n}\left(-\frac{e^{ax}\cos nx}{n}\Big|_{-\pi}^{\pi} + \frac{a}{n}\int_{-\pi}^{\pi} e^{ax}\cos nx\, dx\right)$$

$$\begin{array}{ll|ll} e^{ax} = u & \cos nx\, dx = dv & e^{ax} = u & \sin nx\, dx = dv \\ ae^{ax}dx = du & \dfrac{\sin nx}{n} = v & ae^{ax}dx = du & -\dfrac{\cos nx}{n} = v \end{array}$$

$$I = \frac{a}{n^2}\left(e^{ax}(-1)^n - e^{-ax}(-1)^n\right) - \frac{a^2}{n^2}I$$

$$I\frac{n^2+a^2}{n^2} = \frac{a(-1)^n}{n^2}\left(e^{a\pi} - e^{-a\pi}\right)$$

$$I = \frac{a(-1)^n}{n^2+a^2}\left(e^{a\pi} - e^{-a\pi}\right)$$

$$b_n = \frac{1}{\pi}\underbrace{\int_{-\pi}^{\pi} e^{ax}\sin nx\, dx}_{I_1} = \frac{-n(-1)^n}{\pi(n^2+a^2)}\left(e^{a\pi} - e^{-a\pi}\right)$$

$$I_1 = -e^{ax}\ \frac{\cos nx}{n}\Big|_{-\pi}^{\pi} + \frac{a}{n}\int_{-\pi}^{\pi} e^{ax}\cos nx\, dx = \frac{-1}{n}\left(e^{ax}(-1)^n - e^{-ax}(-1)^n\right) +$$
$$\frac{a}{n}\left(\frac{e^{ax}\sin nx}{n}\Big|_{-\pi}^{\pi} - \frac{a}{n}I_1\right)$$

$$\begin{array}{ll|ll} e^{ax} = u & \sin nx\, dx = dv & e^{ax} = u & \cos nx\, dx = dv \\ ae^{ax}dx = du & -\dfrac{\cos nx}{n} = v & ae^{ax}dx = du & \dfrac{\sin nx}{n} = v \end{array}$$

$$I_1\frac{n^2+a^2}{n^2} = \frac{-(-1)^n}{n}\left(e^{a\pi} - e^{-a\pi}\right)$$

$$I_1 = \frac{-n(-1)^n}{n^2+a^2}\left(e^{a\pi} - e^{-a\pi}\right)$$

$$f(x) = \frac{1}{\pi a2}\left(e^{a\pi} - e^{-a\pi}\right) + \sum_{n=1}^{\infty}\left(\frac{a(-1)^n}{\pi(n^2+a^2)}\left(e^{a\pi} - e^{-a\pi}\right)\cos nx - \frac{n(-1)^n}{\pi(n^2+a^2)}\left(e^{a\pi} - \right.\right.$$
$$\left.\left. e^{-a\pi}\right)\sin nx\right) = \frac{\left(e^{a\pi} - e^{-a\pi}\right)}{\pi}\left(\frac{1}{2a} + \sum_{n=1}^{\infty}\frac{(-1)^n}{(n^2+a^2)}\left(a\cos nx - n\sin nx\right)\right)$$

2. **Funkciju** $f(x) = cos\, ax$ **razviti u Furijeov red na intervalu** $(-\pi, \pi)$.

Rešenje:

$$l = \pi, \qquad f(x) = f(-x) \;\Rightarrow\; b_n = 0$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} cos\, ax\, dx = \frac{2}{\pi}\frac{sin\, ax}{a}\Big|_0^{\pi} = \frac{2}{\pi a}(sin\, a\pi - sin\, 0) = \frac{2sin\, a\pi}{a\pi}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} cos\, ax\, cos\, nx dx = \frac{1}{\pi}\int_0^{\pi}(cos(a+n)x + cos(a-n)x)dx$$

$$= \frac{1}{\pi}\left(\frac{sin(a+n)x}{a+n} + \frac{sin(a-n)x}{a-n}\right)\Big|_0^{\pi} = \frac{1}{\pi}\left(\frac{sin(a+n)\pi}{a+n} + \frac{sin(a-n)\pi}{a-n}\right)$$

$$= \frac{1}{\pi}\left(\frac{sin\, a\pi cos\, n\pi + sin\, n\pi cos\, a\pi}{a+n} + \frac{sin\, a\pi cos\, n\pi - sin\, n\pi cos\, a\pi}{a-n}\right)$$

$$= \frac{1}{\pi}\left(\frac{sin\, a\pi(-1)^n}{a+n} + \frac{sin\, a\pi(-1)^n}{a-n}\right) = \frac{(-1)^n}{\pi}\frac{asin\, a\pi - nsin\, a\pi + asin\, a\pi + nsin\, a\pi}{a^2-n^2}$$

$$= \frac{2a(-1)^n}{\pi(a^2-n^2)}sin\, a\pi$$

$$f(x) = \frac{2sin\, a\pi}{2\pi a} + \sum_{n=1}^{\infty}\frac{2a(-1)^n}{\pi(a^2-n^2)}\, sin\, a\pi\, cos\, nx$$

$$f(x) = \frac{2sin\, a\pi}{\pi}\left(\frac{1}{2a} + \sum_{n=1}^{\infty}\frac{a(-1)^n}{a^2-n^2}\, cos\, nx\right)$$

3. **Funkciju** $f(x) = |x|$ **razviti u Furijeov red na intervalu** $(-1, 1)$.

Rešenje:

$$l = 1, \qquad f(x) = f(-x) \;\Rightarrow\; b_n = 0$$

$$f(x) = \begin{cases} -x, & -1 < x < 0 \\ x, & 0 \le x < 1 \end{cases}$$

$$a_0 = -\int_{-1}^{0} xdx + \int_0^1 xdx = \frac{-x^2}{2}\Big|_{-1}^{0} + \frac{x^2}{2}\Big|_0^1 = \frac{1}{2} + \frac{1}{2} = 1$$

$$a_n = -\int_{-1}^{0} xcos\, n\pi x\, dx + \int_0^1 x\, cos\, n\pi x\, dx$$

$$\begin{array}{ll} x = u & cos\, n\pi x\, dx = dv \\ dx = du & \dfrac{sin\, n\pi x}{n\pi} = v \end{array}$$

$$a_n = -\frac{x\,sin\,n\pi x}{n\pi}\bigg|_{-1}^{0} + \int_{-1}^{0}\frac{sin\,n\pi x}{n\pi}\,dx + \frac{x\,sin\,n\pi x}{n\pi}\bigg|_{0}^{1} - \int_{0}^{1}\frac{sin\,n\pi x}{n\pi}\,dx$$

$$= \frac{-cos\,n\pi x}{n^2\pi^2}\bigg|_{-1}^{0} + \frac{cos\,n\pi x}{n^2\pi^2}\bigg|_{0}^{1} = \frac{-1+(-1)^n}{n^2\pi^2} + \frac{(-1)^n - 1}{n^2\pi^2}$$

$$= \frac{2\big((-1)^n - 1\big)}{n^2\pi^2}$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{2\big((-1)^n - 1\big)}{n^2\pi^2}\,cos\,n\pi x$$

$$f(x) = \frac{1}{2} - \frac{4}{\pi^2}\sum_{n=0}^{\infty}\frac{cos(2n+1)\pi x}{(2n+1)^2}$$

4. Funkciju $f(x) = 10 - x$ razviti u Furijeov red na intervalu $(5, 15)$.

Rešenje:

$l = 5$,

$$a_0 = \frac{1}{5}\int_5^{15}(10-x)dx = \frac{1}{5}\left(10x - \frac{x^2}{2}\right)\bigg|_5^{15} = \frac{1}{5}\left(150 - 50 - \frac{225-25}{2}\right) = 0$$

$$a_n = \frac{1}{5}\int_5^{15} 10\,cos\frac{n\pi x}{5}dx - \frac{1}{5}\int_5^{15} x\,cos\frac{n\pi x}{5}dx = 2\;\frac{sin\frac{n\pi x}{5}}{\frac{n\pi}{5}}\Bigg|_5^{15} -$$

$$x = u \qquad cos\frac{n\pi x}{5}dx = dv$$

$$dx = du \qquad \frac{sin\frac{n\pi x}{5}}{\frac{n\pi}{5}} = v$$

$$\frac{1}{5}\frac{5}{n\pi}\;x sin\frac{n\pi x}{5}\bigg|_5^{15} + \frac{1}{5}\frac{5}{n\pi}\int_5^{15}\sin\frac{n\pi x}{5}dx = \frac{-1}{n\pi}\;\frac{cos\frac{n\pi x}{5}}{\frac{n\pi}{5}}\Bigg|_5^{15} = -\frac{5}{n^2\pi^2}\big(cos\,3n\pi -$$
$$cos\,n\pi\big) = 0$$

$$b_n = \frac{1}{5}\int_5^{15} 10\,sin\frac{n\pi x}{5}dx - \frac{1}{5}\int_5^{15} x\,sin\frac{n\pi x}{5}dx = -2\;\frac{cos\frac{n\pi x}{5}}{\frac{n\pi}{5}}\Bigg|_5^{15} +$$

$$x = u \qquad sin\frac{n\pi x}{5}dx = dv$$

$$dx = du \qquad \frac{-cos\frac{n\pi x}{5}}{\frac{n\pi}{5}} = v$$

$$\frac{1}{5}\frac{5}{n\pi}\;x cos\frac{n\pi x}{5}\bigg|_5^{15} - \frac{1}{5}\frac{5}{n\pi}\int_5^{15}\cos\frac{n\pi x}{5}dx = \frac{1}{n\pi}\big(15(-1)^{3n} - 5(-1)^n\big) - \frac{1}{n\pi}\;\frac{sin\frac{n\pi x}{5}}{\frac{n\pi}{5}}\Bigg|_5^{15} =$$

$\dfrac{10(-1)^n}{n\pi}$

$$f(x)=\sum_{n=1}^{\infty}\frac{10(-1)^n}{n\pi}\,sin\frac{n\pi x}{5}=\frac{10}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\,sin\frac{n\pi x}{5}$$

5. Funkciju $f(x)=5x^2$ razviti u Furijeov red na intervalu $(-\pi,\pi)$.

Rešenje:

$$l=\frac{\pi-(-\pi)}{2}=\pi \qquad f(x)=f(-x)\ \Rightarrow\ b_n=0$$

$$a_0=\frac{2}{\pi}\int_0^{\pi}5x^2dx=\frac{10}{\pi}\frac{x^3}{3}\bigg|_0^{\pi}=\frac{10\pi^2}{3}$$

$$a_n=\frac{5}{\pi}2\int_0^{\pi}x^2cos\frac{n\pi x}{\pi}dx=\frac{10}{\pi}\int_0^{\pi}x^2cosnxdx=\frac{10}{\pi}\left(\frac{x^2sin\,n\pi}{n}\bigg|_0^{\pi}-\frac{2}{n}\int_0^{\pi}x\,sinnxdx\right)$$

$$\begin{array}{ll|ll} x^2=u & cos\,nx\,dx=dv & x=u & sin\,nx\,dx=dv \\ 2xdx=du & \dfrac{sin\,nx}{n}=v & dx=du & -\dfrac{cos\,nx}{n}=v \end{array}$$

$$=-\frac{20}{n\pi}\left(-\frac{xcosnx}{n}\bigg|_0^{\pi}+\frac{1}{n}\int_0^{\pi}cosnxdx\right)$$
$$=-\frac{20}{n\pi}\left(-\frac{\pi cosn\pi-0\cdot cos0}{n}+\frac{sinnx}{n^2}\bigg|_0^{\pi}\right)=\frac{20(-1)^n}{n^2}$$

$$f(x)=\frac{5\pi^2}{3}+\sum_{n=1}^{\infty}\frac{20(-1)^n}{n^2}cos\frac{n\pi x}{\pi}=\frac{5\pi^2}{3}+20\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}cosnx$$

6. Funkciju $f(x)=\dfrac{\pi-x}{3}$ razviti u Furijeov red na intervalu $(0,2\pi)$.

Rešenje: $f(x)=\dfrac{2}{3}\displaystyle\sum_{n=1}^{\infty}\frac{sinax}{n}$

7. Funkciju $f(x)=2x^2-2$ razviti u Furijeov red na intervalu $(-1,1)$.

Rešenje: $f(x)=\dfrac{2}{3}+\dfrac{2}{\pi}\displaystyle\sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2\pi}cosn\pi x+\frac{(-1)^n}{n}sinn\pi x$

8. Funkciju $f(x)=sinax$ razviti u Furijeov red na intervalu $(-\pi,\pi)$.

Rešenje: $f(x)=\dfrac{2sina\pi}{\pi}\displaystyle\sum_{n=1}^{\infty}\frac{n(-1)^n}{a^2-n^2}sinnx$